

Entropy function and universal entropy of two-dimensional extremal black holes

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ABSTRACT: The entropy for two-dimensional black holes is obtained through the entropy function with the condition that the geometry approaches an AdS_2 spacetime in the near horizon limit. It is shown that the entropy is universal and proportional to the value of the dilaton field at the event horizon as expected. We find this universal behavior holds even after the inclusion of higher derivative terms, only modifying the proportional constant. More specifically, a variety of models of the dilaton gravity in two dimensions are considered, in which it is shown that the universal entropy coincides with the well-known results in the previous literatures.

KEYWORDS: Black Holes in String Theory, 2D Gravity.

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1. Introduction

The well-known area law for the black hole entropy that the entropy of black holes is proportional to the area at the horizon might be questionable in two dimensions since the entropy of two-dimensional black hole seems to be zero. In other words, the area of two-dimensional black holes simply vanishes since the hypersurface of the one-dimensional space is a point. Indeed, this is closely related to the fact that the theory of gravity in two dimensions is trivial in the Einstein frame. However, in two-dimensional dilaton gravities [1, 2], it has been shown that the entropy is proportional to the value of the dilaton field at the horizon from the thermodynamic relation and the statistical count of microstates [3–7]. This is supported by the area law in higher-dimensional theory of gravity since the dilaton field is associated with the radius of the compactified coordinate, provided we consider the two-dimensional dilaton gravity as the dimensional reduction from the higher-dimensional theories.

On the other hand, it was shown that the first law of the black hole thermodynamics holds in any theory of gravity derived from the Hamilton-Jacobi's analysis [8], and the generalized definition of the entropy that is simply 2π times the Noether charge at the horizon of the horizon Killing field was suggested [9]. This Noether charge method is useful in studying dynamical black holes and higher derivative theories of gravity since one can no longer use the area formula in those theories. Then, it is plausible to apply this method to two-dimensional black holes, which do not obey the area law. It, however, is not simple to evaluate the entropy from the general form of the two-dimensional dilaton gravity by use of the method due to its complexity.

Recently, it has been conjectured that the macroscopic entropy agrees to the weak-coupling microscopic entropy as long as a certain condition holds, namely, that the geometry near the horizon is given by $AdS_2 \times S^{D-2}$, for all extremal black holes [10]. The entropy

function derived from Wald's formula is defined by, firstly, integrating the Lagrangian density over S^{D-2} , and then taking the Legendre transform of the resulting function with respect to the electric fields with multiplication of an overall factor 2π . The near horizon geometry of the extremal black holes is determined by extremizing the entropy function and the black hole entropy is given by the extremum value of the entropy function. This general method offers an easier way to calculate the black hole entropy in the most of extremal charged black holes.

In this paper, we apply this method to the general form of the two-dimensional scalar-tensor gravity theory, assuming that the near horizon geometry approaches AdS_2 . We show that the (extremal) black hole entropy has a universal form proportional to the value of the dilaton field at the horizon. We also find that the model, after the inclusion of higher derivative terms, admit the solution which has AdS_2 geometry with $SO(2, 1)$ isometry. In this case, therefore, we find that the universal behavior holds even after the inclusion of higher derivative terms, modifying only the overall proportionality constant.

In section 2, we obtain the universal behavior of the extremal black hole entropy for the generic two-dimensional scalar-tensor gravity with the gauge field. We also consider the possible higher-derivative corrections in the action and find their implications in the black hole entropy. In section 3, we apply the method to the two-dimensional gravity which is a low energy effective theory of the heterotic strings. In section 4, we consider the Jackiw-Teitelboim model with the extra gauge field. In section 5, we obtain the black hole entropy in two-dimensional doubly charged CGHS model. In section 6, we draw some discussions.

2. Universal entropy in two-dimensional extremal black holes

We now calculate the entropy function for two-dimensional scalar-tensor gravity. A general form of the two-dimensional scalar-tensor gravity action coupled to a gauge field strength $F_{\mu\nu}$ is given by

$$I = \frac{1}{2\pi} \int d^2x \sqrt{-g} \Phi \left[R + T(\Phi)(\nabla\Phi)^2 + \lambda^2 U(\Phi) - \frac{1}{4} V(\Phi) F^2 \right], \quad (2.1)$$

where $T(\Phi)$, $U(\Phi)$, and $V(\Phi)$ are arbitrary functions with respect to a scalar field Φ , and Λ is a two-dimensional cosmological constant. In many cases this two-dimensional theory comes from the dimensional reduction of the higher-dimensional theories. The theory may also admit charged black hole solutions, including extremal black hole solutions. We would like to find out the general form of the entropy of these extremal black holes following the procedure in ref. [10]. It is, therefore, natural to assume that the extremal black hole solutions reduces AdS_2 in the near horizon limit, with $SO(2, 1)$ symmetry. Then, the near horizon solution of the extremal black hole with charge q can be, generically, written in

the form of

$$\begin{aligned}
 ds^2 &= v \left(-r^2 dt^2 + \frac{dr^2}{r^2} \right), \\
 \Phi &= u, \\
 F_{rt} &= \varepsilon,
 \end{aligned} \tag{2.2}$$

where v , u , and ε are constants to be determined in terms of the charge q and the cosmological constant Λ . Note that the covariant derivatives of the Riemann tensor, the scalar field, and the gauge field strength all vanish in this near horizon geometry, which plays an important role to construct Sen's entropy function from Wald's entropy formula. For these field configurations, the Lagrangian density becomes a function of v , u , and ε ;

$$\begin{aligned}
 f(v, u, \varepsilon) &= \frac{1}{2\pi} \sqrt{-g} \Phi \left[R + T(\Phi)(\nabla\Phi)^2 + \lambda^2 U(\Phi) - \frac{1}{4} V(\Phi) F^2 \right] \\
 &= \frac{vu}{2\pi} \left[-\frac{2}{v} + \lambda^2 U(u) + \frac{V(u)}{2v^2} \varepsilon^2 \right],
 \end{aligned} \tag{2.3}$$

and from which one finds the electric charge as

$$q = \frac{\partial f}{\partial \varepsilon} = \frac{uV(u)}{2\pi v} \varepsilon. \tag{2.4}$$

Now, the entropy function [10] is defined as the Legendre transform of the Lagrangian density with respect to the gauge field ε ,

$$\begin{aligned}
 F(v, u, q) &= 2\pi[q\varepsilon - f(v, u, \varepsilon)] \\
 &= vu \left[\frac{2}{v} - \lambda^2 U(u) + \frac{2\pi^2 q^2}{u^2 V(u)} \right].
 \end{aligned} \tag{2.5}$$

The undetermined parameter u, v can be fixed by the equations of motion, which becomes the extremum equations as

$$\left. \frac{\partial F}{\partial v} \right|_{(u_e, v_e)} = u_e \left[-\lambda^2 U(u_e) + \frac{2\pi^2 q^2}{u_e^2 V(u_e)} \right] = 0, \tag{2.6}$$

$$\left. \frac{\partial F}{\partial u} \right|_{(u_e, v_e)} = \left[2 - \lambda^2 v_e (U(u_e) + u_e U'(u_e)) - \frac{2\pi^2 v_e q^2}{u_e^2 V^2(u_e)} (V(u_e) + u_e V'(u_e)) \right] = 0, \tag{2.7}$$

where the prime (\prime) denotes the derivative with respect to u . The entropy is given by the value of the entropy function at the extremum,

$$S_{\text{BH}}(q) = F(v_e, u_e, q) = 2u_e \tag{2.8}$$

from eqs. (2.5) and (2.6). Note that the entropy is proportional to the overall 'effective' coupling constant associated with the dilaton field, u , as is expected. The effective Newton constant in two dimensions is given by $\kappa^2 = 2\pi/\langle\Phi\rangle$ where $\langle\Phi\rangle$ is an expectation value of the dilaton field. In this set up, it is natural to take the value of the dilaton field at the horizon, and hence the effective two-dimensional Newton constant becomes $\kappa^2 = 2\pi/u_e$. In

this sense, this is a universal result in that it depends only on the overall coupling constant of the Lagrangian, irrespective of the specific form of the action.¹ The alternative form of the black hole entropy, which will be useful later on, is given by

$$S_{\text{BH}}(q) = \frac{2\pi^2 v q^2}{V} [\ln(u^2 UV)]' \Big|_{(u_e, v_e)} \quad (2.9)$$

Now let us consider the effect of higher derivative terms. Since in two dimensions Riemann tensor and Ricci tensor can be expressed in terms of Ricci scalar, it is sufficient to consider the higher derivative terms of the form R^n . If one includes the generic higher derivative terms with additional terms in the action, $\Delta I = \frac{1}{2\pi} \int d^2x \sqrt{-g} \Phi [\sum a_n R^n]$, the change in the entropy function is given by $\Delta F = -u \sum b_n v^{1-n}$, where $b_n = (-2)^n a_n$. Then, the entropy (2.8) is modified as

$$S_{\text{mod}} = u_e [2 - \sum n b_n v_e^{1-n}]. \quad (2.10)$$

The exact expression and implication of the black hole entropy S_{BH} will be given through several interesting examples in what follows.

3. Two-dimensional effective heterotic string theory

First, we consider the two-dimensional gravity, which may come from the compactification of the heterotic string theory, with the dilaton field, $U(1)$ gauge field and the two-dimensional cosmological constant. The action is given by

$$I = \frac{1}{2\pi} \int d^2x \sqrt{-g} e^{-2\phi} \left[R + 4(\nabla\phi)^2 + 4\lambda^2 - \frac{1}{4}F^2 \right]. \quad (3.1)$$

This theory was used in the Callan-Giddings-Harvey-Strominger (CGHS) model [1] to study the quantum nature of the black holes. It was known that the theory admits the static charged black hole solutions of the form [11]

$$\begin{aligned} ds^2 &= -g(r)dt^2 + \frac{1}{g(r)}dr^2, \\ \phi &= -\lambda r, \\ F_{rt} &= \sqrt{2}Qe^{-2\lambda r}, \end{aligned} \quad (3.2)$$

where $g(r) = 1 - (M/\lambda)e^{-2\lambda r} + (Q^2/4\lambda^2)e^{-4\lambda r}$. In the extremal limit, $M = Q$, the metric function is reduced to $g(r) = [1 - (Q/2\lambda)e^{-2\lambda r}]^2$ and the near horizon geometry in the leading order is clearly given by AdS_2 type. The near horizon solutions which has $SO(2,1)$ symmetry can be written as eq. (2.2) with $\Phi = e^{-2\phi}$. In this case the Lagrangian density function is evaluated as

$$\begin{aligned} f(v, u, \varepsilon) &= \frac{1}{2\pi} \sqrt{-g} e^{-2\phi} \left[R + 4(\nabla\phi)^2 + 4\lambda^2 - \frac{1}{4}F^2 \right] \\ &= \frac{vu}{2\pi} \left[-\frac{2}{v} + 4\lambda^2 + \frac{1}{2v^2}\varepsilon^2 \right], \end{aligned} \quad (3.3)$$

¹We find the overall factor difference in black hole entropy between our results and the results in the literatures by 2π or π . This is purely due to the convention of two-dimensional Newton constant.

where the electric charge is given by

$$q = \frac{\partial f}{\partial \varepsilon} = \frac{u}{2\pi v} \varepsilon. \quad (3.4)$$

Then, the entropy function is written as

$$F = 2\pi[q\varepsilon - f] = vu \left[\frac{2}{v} - 4\lambda^2 + \frac{2\pi^2 q^2}{u^2} \right], \quad (3.5)$$

and extremizing eq. (3.5) with respect to v and u ,

$$\left. \frac{\partial F}{\partial v} \right|_{(u_e, v_e)} = \left[-4\lambda^2 u_e + \frac{2\pi^2 q^2}{u_e} \right] = 0, \quad (3.6)$$

$$\left. \frac{\partial F}{\partial u} \right|_{(u_e, v_e)} = \left[2 - 4\lambda^2 v_e - \frac{2\pi^2 v_e q^2}{u_e^2} \right] = 0. \quad (3.7)$$

yields solutions $u_e^2 = \pi^2 q^2 / 2\lambda^2$ and $v_e = 1/4\lambda^2$ in terms of the black hole charge, q and the cosmological constant, λ . In fact, these are consistent with the near horizon limit of the extremal black holes in eq. (3.2) with $M = Q$ as it should be. Note that, in taking the near horizon limit we should make replacements, $4\lambda^2(r - r_H) \rightarrow r$ and $q = \frac{Q}{\sqrt{2\pi}}$.

Plugging these values u_e and v_e into the entropy function, one finds the black hole entropy

$$S_{\text{BH}} = 2u_e = \frac{\sqrt{2\pi}|q|}{\lambda}, \quad (3.8)$$

which agrees to the result in ref. [3].

Now, considering generic higher-derivative correction terms, the extremizing equations (3.6) and (3.7) in the CGHS-Maxwell model are modified as

$$\left. \frac{\partial F}{\partial v} \right|_{(u_e, v_e)} = \left[-4\lambda^2 u_e + \frac{4\pi^2 q^2}{2u_e} - u_e \sum_{n=2} b_n (1-n) v_e^{-n} \right] = 0, \quad (3.9)$$

$$\left. \frac{\partial F}{\partial u} \right|_{(u_e, v_e)} = \left[2 - 4\lambda^2 v_e - \frac{4\pi^2 v_e q^2}{2u_e^2} - \sum_{n=2} b_n v_e^{1-n} \right] = 0. \quad (3.10)$$

Using these equations, the entropy (2.10) can be written in terms of q , $S_{\text{mod}} = 4\pi^2 v_e q^2 / u_e$. In this two-dimensional set-up, the only dimensionful geometric quantity is λ . Checking the dimension of total action $I + \Delta I$, we see that the coefficient a_n has the dimension of order $\lambda^{2(1-n)}$, and it is straightforward to check that u is of order q/λ and v is of order λ^{-2} from eqs. (3.9) and (3.10). Then, it is easily seen that the modified entropy is of order q/λ , which yields that higher-derivative correction terms only modify the factor of the original entropy (3.8) and can be written generically as $S_{\text{BH}} = a \frac{|q|}{\lambda}$, where a is the pure numerical factor depending on the higher derivative terms.

For example, one may consider four-dimensional Gauss-Bonnet terms, which appear in the low energy effective theory of heterotic strings, of the form

$$\Delta I = \frac{1}{16\pi} \int d^4x \sqrt{-g_{(4)}} e^{-2\phi_{(4)}} \{ R_{(4)\mu\nu\rho\sigma} R_{(4)}^{\mu\nu\rho\sigma} - 4R_{(4)\mu\nu} R_{(4)}^{\mu\nu} + R_{(4)}^2 \}. \quad (3.11)$$

If the four-dimensional metric can be written as the direct product of two-dimensional space and two-sphere, S^2 , with constant radius, the two-dimensional effective action after the dimensional reduction becomes

$$\Delta I = \frac{\alpha}{2\pi} \int d^2x \sqrt{-g} e^{-2\phi} R \quad (3.12)$$

where the numerical constant α is related to the ratio of the four-dimensional Newton constant and the volume of two-sphere. Therefore the incorporation of the four-dimensional Gauss-Bonnet terms change only the two-dimensional Newton constant, and thus the AdS geometry becomes the exact solution of the full theory. After including these higher-derivative terms, the black hole entropy gets modified and becomes

$$S_{\text{BH}} = 2(1 + \alpha)u_e = (1 + \alpha) \frac{\sqrt{2\pi}|q|}{\lambda}. \quad (3.13)$$

4. JT model

Another interesting model in two-dimensional gravity is the Jackiw-Teitelboim (JT) model [2]. In order to study the extremal charged black hole, one can include a gauge field in the model with the following form of the action

$$I = \frac{1}{2\pi} \int d^2x \sqrt{-g} e^{-2\phi} \left[R + 4\lambda^2 - \frac{1}{4} e^{-4\phi} F^2 \right]. \quad (4.1)$$

Again, one may regard this model as the dimensional reduction of a higher-dimensional gravity. Indeed the action can be obtained from the Kaluza-Klein (KK) compactification along the circle direction of the three-dimensional Einstein-Hilbert action with a cosmological constant. It was known that the model also admits the static charged solutions whose configurations are given by

$$\begin{aligned} ds^2 &= -g(r)dt^2 + \frac{1}{g(r)}dr^2, \\ e^{-2\phi} &= \sqrt{2}\lambda r, \\ F_{rt} &= \frac{Q}{2\lambda^3 r^3}, \end{aligned} \quad (4.2)$$

where $g(r) = 2\lambda^2 r^2 - M/\lambda + Q^2/8\lambda^4 r^2$ [12]. In the extremal limit, the metric function is reduced to $g(r) = 2\lambda^2 r^2 [1 - Q/4\lambda^3 r^2]^2$. The near horizon solutions are those of AdS_2 with $SO(2,1)$ symmetry. Therefore one can study the black hole entropy in the similar fashion.

The entropy function, F , which is the Legendre transform of the Lagrangian density function, f , in terms of the electric field becomes

$$F = 2\pi[q\varepsilon - f] = vu \left[\frac{2}{v} - 4\lambda^2 + \frac{2\pi^2 q^2}{u^4} \right], \quad (4.3)$$

where the electric charge can be determined as $q = \partial f / \partial \varepsilon = u^3 \varepsilon / 2\pi v$. One may see that with the action given above, i.e. without higher derivative terms, the charge q is related to the parameter Q of the black hole solutions as $q = Q/\sqrt{2}\pi$.

Extremizing it with respect to v and u ,

$$\left. \frac{\partial F}{\partial v} \right|_{(u_e, v_e)} = \left[-4\lambda^2 u_e + \frac{2\pi^2 q^2}{u_e^3} \right] = 0, \tag{4.4}$$

$$\left. \frac{\partial F}{\partial u} \right|_{(u_e, v_e)} = \left[2 - 4\lambda^2 v_e - \frac{6\pi^2 v_e q^2}{u_e^4} \right] = 0. \tag{4.5}$$

provides the extremizing solutions $u_e^4 = \pi^2 q^2 / 2\lambda^2$ and $v_e = 1/8\lambda^2$. By plugging these back into the entropy function, we obtain the black hole entropy as

$$S_{\text{BH}} = 2u_e = 2^{3/4} \sqrt{\frac{\pi|q|}{\lambda}}, \tag{4.6}$$

which is exactly the same result as in ref. [4] except for the extra $1/\lambda$ factor.² One may also consider the higher derivative corrections along the similar fashion. It will give a numerical corrections to an overall factor as discussed in the previous section.

5. Doubly charged CGHS model

Finally, we consider the doubly charged CGHS model which can be obtained by the dimensional reduction from the ten-dimensional IIB supergravity action coupled to the two-form Ramond-Ramond gauge field. One may consider the type IIB black holes on $M_5 \times S^1 \times T^4$, where D5-brane wraps the five torus $S^1 \times T^4$ Q_5 times, D strings wrap the circle S^1 Q_2 times, and a KK momentum Q runs along the circle S^1 [7, 13]. Compactifying M_5 to $M_2 \times S^3$, the reduced two-dimensional action is given by

$$I = \frac{1}{2\pi} \int d^2x \sqrt{-g} e^{-2\phi} \left[R + 4(\nabla\phi)^2 + 4\lambda^2 - (\nabla\psi)^2 - \frac{1}{4}(\nabla\psi_1)^2 - \frac{1}{4}e^{-\psi_1} F_2^2 - \frac{1}{4}e^{\psi_1} F^2 \right], \tag{5.1}$$

where the cosmological constant λ is related to the radius of S^3 , $\lambda = e^{\psi_2}$, which is assumed to be constant. The field strength $F = dA$ has the KK momentum as its charge and the field strength $F_{2\alpha\beta} = H_{\alpha\beta x^5}$ comes from the string wrapping along the circle S^1 .

The static solutions are given by

$$ds^2 = -\frac{\beta^2(1 - r_0^2/r^2)}{(1 + r_1^2/r^2)(1 + r_2^2/r^2)} dt^2 + \frac{dr^2}{\lambda^2 r^2 (1 - r_0^2/r^2)}, \tag{5.2}$$

$$e^{-2\phi} = \frac{\sqrt{(r^2 + r_1^2)(r^2 + r_2^2)}}{\lambda}, \tag{5.3}$$

$$F_{rt} = \frac{2rQ\beta}{(r^2 + r_2^2)^2}, \tag{5.4}$$

$$F_{2rt} = \frac{2rQ_2\beta}{(r^2 + r_1^2)^2}, \tag{5.5}$$

where $r_1^2 = \sqrt{Q_2^2 + r_0^4/4} - r_0^2/2$ and $r_2^2 = \sqrt{Q^2 + r_0^4/4} - r_0^2/2$ are constant, and $r_0 \rightarrow 0$ in the extremal limit [13]. The near horizon geometry of the extremal charged black hole

²Note that the entropy (4.6) is dimensionless in mass dimension, while that of ref. [4] has length dimension.

solution becomes AdS_2 and the field configurations become symmetric under the $SO(2, 1)$. Generically, the field configurations which are consistent with $SO(2, 1)$ are given by eq. (2.2) for the metric, dilaton, and one gauge field along with the following field configurations for the other two scalars and extra gauge field as

$$\begin{aligned}\psi &= u_\psi, \\ \psi_1 &= \ln u_{\psi_1}, \\ F_{2\bar{r}t} &= \varepsilon_2.\end{aligned}\tag{5.6}$$

The Lagrangian density function is, then, given by

$$f(v, u, \varepsilon) = \frac{vu}{2\pi} \left[-\frac{2}{v} + 4\lambda^2 + \frac{u_{\psi_1}}{2v^2} \varepsilon^2 + \frac{1}{2v^2 u_{\psi_1}} \varepsilon_2^2 \right],\tag{5.7}$$

from which one can determine the electric charges of the black hole as

$$q = \frac{uu_{\psi_1}}{2\pi v} \varepsilon, \quad q_2 = \frac{u}{2\pi v u_{\psi_1}} \varepsilon_2.\tag{5.8}$$

The entropy function becomes

$$F = vu \left[\frac{2}{v} - 4\lambda^2 + \frac{2\pi^2 q^2}{u^2 u_{\psi_1}} + \frac{2\pi^2 u_{\psi_1} q_2^2}{u^2} \right],\tag{5.9}$$

whose extremum is determined by

$$\left. \frac{\partial F}{\partial v} \right|_{(u_e, v_e, u_{e\psi_1})} = \left[-4\lambda^2 u_e + \frac{2\pi^2 q^2}{u_e u_{e\psi_1}} + \frac{2\pi^2 u_{e\psi_1} q_2^2}{u_e} \right] = 0,\tag{5.10}$$

$$\left. \frac{\partial F}{\partial u} \right|_{(u_e, v_e, u_{e\psi_1})} = \left[2 - 4\lambda^2 v_e - \frac{2\pi^2 v_e q^2}{u_e^2 u_{e\psi_1}} - \frac{2\pi^2 v_e u_{e\psi_1} q_2^2}{u_e^2} \right] = 0,\tag{5.11}$$

$$\left. \frac{\partial F}{\partial u_{\psi_1}} \right|_{(u_e, v_e, u_{e\psi_1})} = \left(\frac{4\pi^2 v_e}{2u_e} \right) \left[- \left(\frac{q^2}{u_{e\psi_1}^2} \right) + q_2^2 \right] = 0.\tag{5.12}$$

The extremizing solutions are

$$u_e^2 = \frac{\pi^2 |qq_2|}{\lambda^2}, \quad v_e = \frac{1}{4\lambda^2}, \quad u_{e\psi_1}^2 = \frac{q^2}{q_2^2}.\tag{5.13}$$

Note that with the help of the extremizing solutions (5.13), one can determine the relations between the black hole charges (q, q_2) and the parameters of the black hole solutions (Q, Q_2) as $q = Q/\pi$ and $q_2 = Q_2/\pi$. Plugging the extremum values (5.13) into the entropy function finally yields the black hole entropy as

$$S_{\text{BH}} = 2u_e = \frac{2\pi \sqrt{|qq_2|}}{\lambda},\tag{5.14}$$

which is exactly the same result as in ref. [7]. It is straightforward to include the higher derivative terms and find out the corrections to the entropy. There are another extremal black hole solutions in a non-perturbatively defined string theory of the two-dimensional type 0 string, which has the matrix model description [14, 15]. They also have the near horizon geometry of AdS_2 , but we will not consider them in this paper.

6. Discussions

For black holes with supersymmetry, a successful statistical understanding of the thermodynamic Bekenstein-Hawking entropy has been achieved by counting microstates [16]. This resulted from the BPS property of supersymmetric black holes - the BPS states can be computed at weak coupling regime and then analytically continued to the strong coupling regime. On the other hand, the attractor mechanism that the macroscopic entropy is independent of the asymptotic values of the moduli is closely related to the near horizon geometry of extremal black holes. Both significant properties simplified the issue of evaluating the entropy of supersymmetric black holes and offered the agreement between the thermodynamic and the statistical entropies. Motivated by the fact that there are many indications of this agreement for non-supersymmetric black holes, it has been conjectured that the same agreement still work and the origin does not rely on supersymmetry but the attractor mechanism of all extremal black holes whether or not they are supersymmetric [17]. More precisely, the thermodynamic Bekenstein-Hawking entropy of all extremal black holes within string theory, in particular, showing a near-horizon geometry of $AdS_2 \times S^{D-2}$ exactly matches with the statistical microscopic entropy at weak coupling regime. As seen in ref. [17], there are a variety of examples of non-supersymmetric black holes that support the surmised agreement between the macroscopic and the microscopic entropies for all extremal black holes.

In this paper, using the Sen's entropy function, we found the universal behavior of the black hole entropy in the two-dimensional gravity. We also considered the higher derivative corrections and found the universal behavior remains to hold. In all cases we considered, the extremal black hole entropy only depends on the field configuration at the horizon, regardless of its asymptotic values of the fields. In this sense, these examples also show the attractor mechanism of the black hole entropy [18–20]. Hence, our studies on the entropy function method of two-dimensional black holes might be another good examples of finding the entropy of non-supersymmetric black holes, which robustly upholds the underlying conjecture.

One motivation to study the black hole entropy in the context of the two-dimensional gravity is to probe the quantum nature of gravity. In the two-dimensional gravity, the quantum corrections of the scalar field typically include Polyakov-Liouville (PL) action [21, 22]. In fact, we considered this issue and found that the entropy function method may not be applied to two-dimensional quantum-corrected model because the nonlocality in PL term produces an inconsistent extremal equations. It seems to imply that the near horizon geometry is no longer AdS_2 type after the quantum corrections. Even if the near horizon solution is reduced to AdS_2 , there remains another problem. Localizing PL action, by introducing an auxiliary field, say Ψ , requires that $\square\Psi = R$. In the near horizon limit, the equation is written as $\partial_{\tilde{r}}\tilde{r}^2\partial_{\tilde{r}}\Psi = -2$, and the solution, $\Psi = -2\ln\tilde{r} + c_1/\tilde{r} + c_2$, is divergent at the horizon, which violates the assumption that the scalar field near horizon has a finite constant value. Further studies on this issue is needed.

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